

On the Fairness of Additive Welfarist Rules

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Fair Division

- We allocate **indivisible resources** among a group of agents.
- How to ensure **fairness** and **efficiency**?
- Example: inheritance.



Definitions

- An **instance** consists of:
 - A set N of **agents** $1, \dots, n$.
 - A set M of **goods** g_1, \dots, g_m which are **indivisible**.
 - Each agent i **values**:
 - good g **non-negatively** at $u_i(g) \geq 0$;
 - bundle of goods **additively** i.e.
$$u_i(\{g_1, \dots, g_k\}) = u_i(g_1) + \dots + u_i(g_k).$$
- An **allocation** (A_1, \dots, A_n) is an ordered partition of the goods M .

Definitions (cont'd)

- **Fairness**

- **Envy-free up to 1 good (EF1)**: For all i, j , either $A_j = \emptyset$ or there is a $g \in A_j$ with

$$u_i(A_i) \geq u_i(A_j \setminus \{g\}).$$

- **Efficiency**

- **Additive welfarist rule** with function f (which is strictly increasing): Chooses an allocation that maximizes

$$\sum_{i=1}^n f(u_i(A_i)).$$

- **Examples:**

- **Maximum utilitarian welfare (MUW)**: $f(x) = x$.
- **Maximum Nash welfare (MNW)**: $f(x) = \log x$.
- **Maximum harmonic welfare (MHW)**: $f(x) = \sum_{k=1}^x 1/k$.

Past Work

- Real-valued:
 - MNW guarantees EF1 [Caragiannis et al., 2019].
 - MNW is the only additive welfarist rule which guarantees EF1 for all real-valued instances [Suksompong, 2023].
- Integer-valued:
 - MHW guarantees EF1 for all integer-valued instances [Montanari et al., 2024].
 - An important subclass in terms of real world application.
 - Example: Spliddit¹.

Question

For each subclass of instances (e.g. integer-valued, two-value), which additive welfarist rules guarantee EF1?

¹<http://www.spliddit.org/>

Motivating Example

	g_1	g_2	g_3	\dots	g_n
Agent 1	n				
Agent 2	$n-1$	1			
Agent 3		$n-1$	1		
\vdots			\dots	\dots	
Agent n				$n-1$	1

MNW: Agent 1 gets value n and every other agent gets value 1.

MHW: Agent 2 gets value 0 and every other agent gets value $\geq n - 1$.

MHW allocation is arguably **better** than MNW allocation when n is large.

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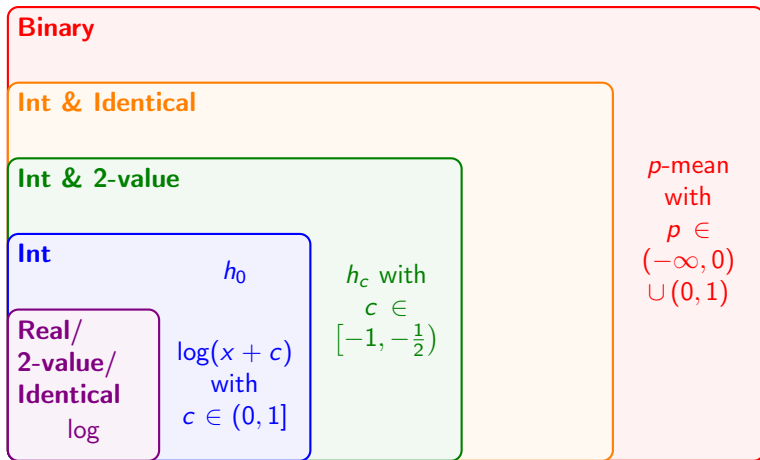
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Our Contribution

- **Real-valued:**
 - **No restriction:** MNW is the only additive welfarist rule guaranteeing EF1 [Caragiannis et al., 2019; Suksompong, 2023].
 - We strengthen this result to more restricted subclasses:
 - **identical-good** instances, and
 - **two-value** instances.
- **Integer-valued:**
 - There are infinitely many other additive welfarist rules guaranteeing EF1.
 - Example: $f(x) = \log(x + c)$ for any $c \in [0, 1]$.
 - We give necessary and sufficient conditions.

Our Contribution



Note: $h_c(x) = \sum_{t=1}^x \frac{1}{t+c}$ is the “modified harmonic number”.

Future Work

- **Other settings:**
 - Non-additive welfarist rules
 - Other fairness notions (e.g. PROP1)
- **Comparison:**
 - For some subclasses (e.g. integer-valued instances), there are many additive welfarist rules that yield EF1.
 - **Which rule is the “best”?**
 - Method of comparison: e.g. price of fairness [Bei et al., 2021; Celine et al., 2023; Li et al., 2024].
- **Computation:** How to find allocations satisfying additive welfarist rules other than MNW?